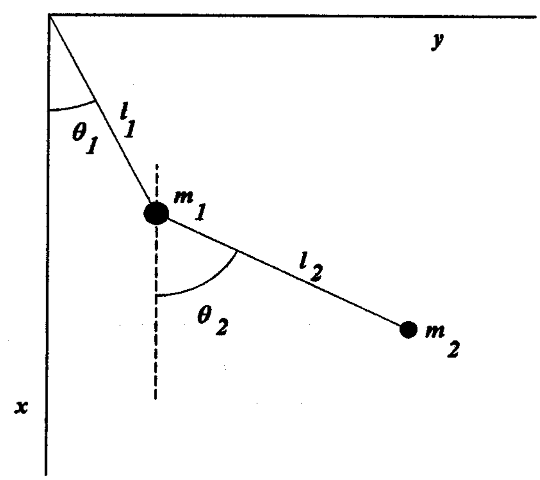
Physics Graduate School Qualifying Examination

17 August 2000

Part I

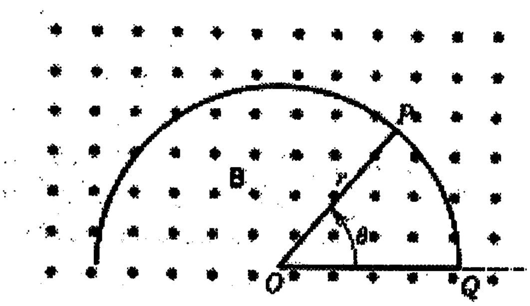
Instructions: Work all problems. This is a closed book examination. Calculators may not be used. **Start each problem on a new pack of yellow paper and use only one side of each sheet. Write your 3-digit student number on the upper right hand corner of each and every answer sheet, even if blank. All sheets which you receive should be handed in, even if blank.** All problems carry the same weight.

I-1. A double pendulum consists of two simple pendula, with one pendulum suspended from the bob of the other, as drawn below. If both pendula move in the same plane, determine the Lagrangian when their masses as well as lengths are not equal, $m_1 \neq m_2$ and $l_1 \neq l_2$. Use angles as generalized coordinates in the Lagrangian but do not assume small angles.



I-2. Two masses, m_1 and m_2, are connected by a massless ideal spring with spring constant k and can move without friction on a horizontal rod. At initial moment, $t=0$, the string is in its uncompressed and unstretched state, but the masses have velocities $v_1(0)$ and $v_2(0)$, respectively (which are known). Find the velocities of the masses at all subsequent times.

I-3. A wire with cross sectional area A and resistivity ρ is bent into a circular arc of radius r as shown below. An additional straight length of this wire, OP, is free to pivot about O and makes sliding contact with the arc at P. Finally, another straight length of this wire, OQ, completes the circuit. The entire arrangement is located in a magnetic field of magnitude B directed out of the plane of the figure. The straight wire OP starts from rest with $\theta = 0$ and has a constant angular acceleration of α.



I-3.) 1.) The resistance of the loop $OPQO$ as a function of θ is given by

a) R

c) $\frac{\rho r \theta}{A}$

b) $\frac{\rho r}{A}(2+\theta)$

d) $\frac{2\rho r}{A}$

I-3.) 2.) The magnetic flux through the loop as a function of θ is

a) $\Phi_B = \frac{1}{2} B r^2 \theta$

c) $\Phi_B = rBr\theta$

b) $\Phi_B = BA$

d) $\Phi_B = \frac{1}{2} B \pi r^2$

I-3.) 3.) The induced current, i, in the loop is

a) $i = \frac{\mathcal{E}}{R}$

c) $i = \frac{1}{\sqrt{2}} \frac{ABr}{\rho(2+\theta)} \sqrt{\alpha \theta}$

b) $i = R\alpha t$

d) $i = \frac{1}{2} r^2 B \alpha t$

I-3.) 4.) For what value of θ is the induced current in the loop a maximum?

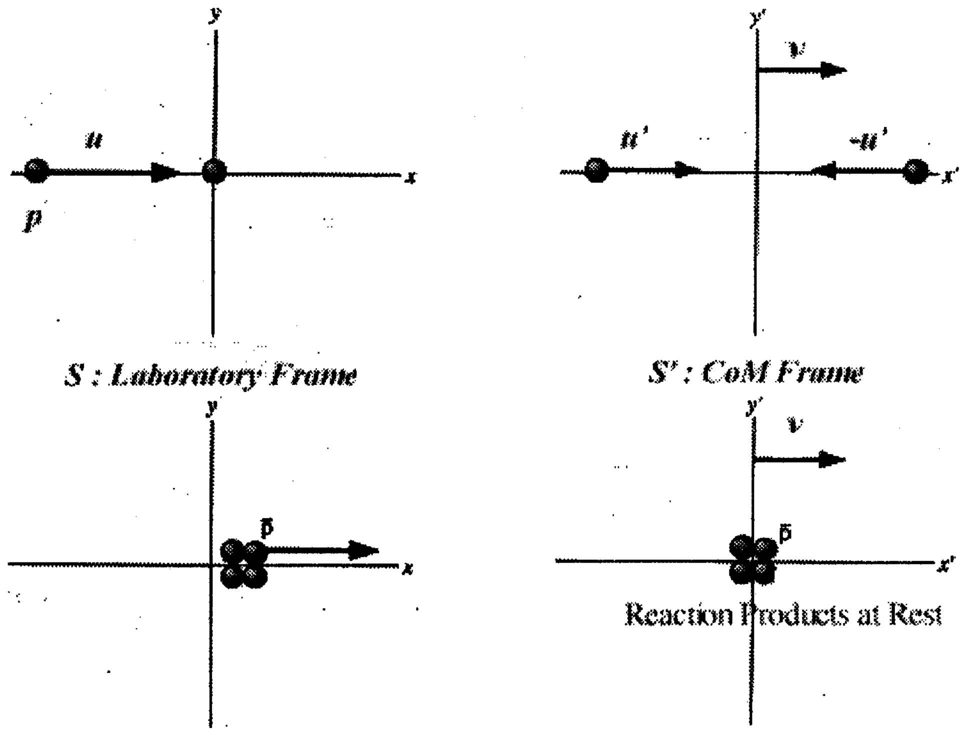
I-4. Calculate the inductance per unit length of a system of two parallel superconducting wires carrying currents of equal magnitude I in opposite directions (consider this as a rectangular loop with two of the sides long parallel wires). Each wire has a circular cross section with radius a, and their central axes are a distance $d>2a$ apart. (In a superconducting wire, the current is confined to a thin boundary layer.)

I-5. According to Fermi statistics, no two indistinguishable Fermions can occupy the same state, while in Bose statistics arbitrarily many indistinguishable Bosons can occupy the same state. What is the difference in ground state energy of a system of five, mass m, spin $3/2$ Fermion particles confined to a one-dimensional box of width a, versus five, mass m, spin 1 Boson particles in the same box? Note there are no spin dependent terms in the energies.

I-6. The discovery of the antiproton $\bar{p}$ (a particle with the same rest energy as a proton, $m_p c^2 = 938$ MeV, but with the opposite electric charge) took place in 1956 through the following reaction:

$$p + p \rightarrow p + p + p + \bar{p}$$

in which accelerated protons were incident on a target of protons at rest in the laboratory (reference frame of observer S, see the diagram below). The minimum incident kinetic energy needed to produce the reaction is called the threshold kinetic energy, for which the final particles move together as if they were a single unit. From the center of momentum reference frame (observer S'), in which the incident proton and antiproton move with equal and opposite velocities and the reaction products are produced at rest, the reaction appears as drawn below.



I-6.) 1.) Using energy conservation determine the speed v of observer S' with respect to observer S.

a) $v = \sqrt{\frac{15}{16}}\,c$

b) $v = \sqrt{\frac{3}{4}}\,c$

c) $v = \sqrt{\frac{1}{2}}\,c$

d) $v = \frac{4\sqrt{3}}{7}\,c$

I-6.) 2.) What is the momentum of the proton in the center of momemtum (S') frame?

a) $p'c = \sqrt{15}\,m_p c^2$

b) $p'c = \sqrt{3}\,m_p c^2$

c) $p'c = m_p c^2$

d) $p'c = 4\sqrt{3}\,m_p c^2$

I-6.) 3.) Determine the threshold kinetic energy to produce antiprotons in this reaction.

a) $K = 22\ m_p c^2$

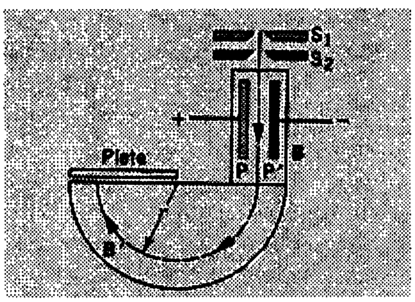
b) $K = 6\ m_p c^2$

c) $K = 2\sqrt{2}\ m_p c^2$

d) $K = 61\ m_p c^2$

I-7. A photon with circular polarization can have angular momentum $\pm\hbar$. A ball of mass M, radius R, and uniform mass density is initially motionless. After absorption of W watts of wavelength λ light with counter clockwise circular polarization for a time t, where the light is incident from the bottom of the ball on the symmetry axis of the ball from the $-z$ direction toward the $+z$ direction, what is the ball's angular rotation velocity ω?

I-8. Bainbridge's mass spectrometer, shown below, separates ions having the same velocity. The ions, after entering through slits S_1 and S_2, pass through a velocity selector composed of an electric field produced by the charged plates P and P', and a magnetic field **B** perpendicular to the electric field and the ion path. Those ions that pass undeviated through the crossed **E** and **B** fields enter into a region where a second magnetic field **B'** exists, and are bent into circular paths. A photographic plate registers their arrival. Find the expression for the charge to mass ratio, q/m, of ions with radius r of their circular orbit.



Physics Graduate School Qualifying Examination

18 August 2000

Part II

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II-1. In this problem we will use cylindrical (ρ, ϕ, z) and spherical (r, θ, ϕ) coordinates, defined as follows:

Cylindrical: $x = \rho\cos(\phi)$, $y = \rho\sin(\phi)$, $z = z$

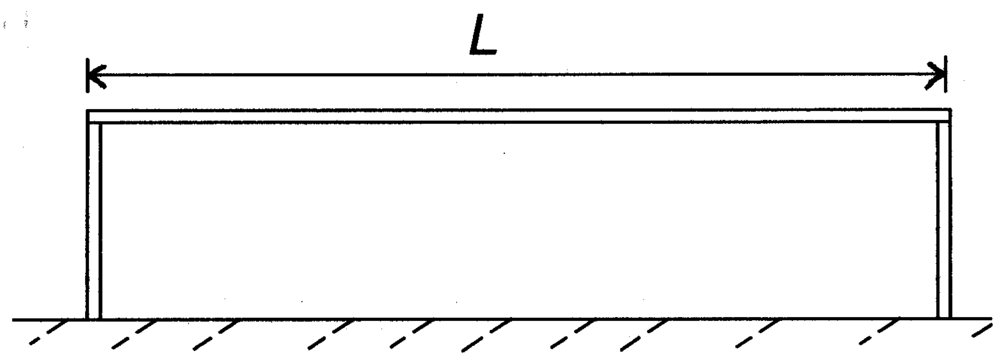
Spherical: $x = r\sin(\theta)\cos(\phi)$, $y = r\sin(\theta)\sin(\phi)$, $z = r\cos(\theta)$

a) For a particle of mass M, write the classical (i.e. nonrelativistic) kinetic energy in terms of cylindrical coordinates.

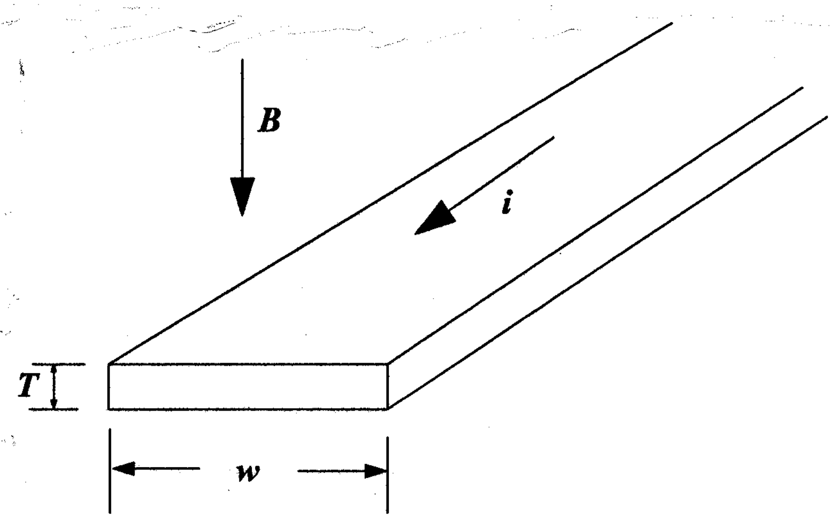
b) For a particle of mass M, write the clasical kinetic energy in terms of spherical coordinates.

c) For a particle of mass M subject to a potential $V(r)$, derive the three equations of motion in spherical coordinates.

II-2. A uniform thin rigid rod of weight W and length L is supported in a horizontal position by two props at its ends as shown. At $t = 0$, one prop is kicked out of position. Find the force on the other prop at that time.



II-3. The Hall effect can be used to measure the number density and sign of the charge carriers. Show how a measurement of the magnetic field perpendicular to a width w of the conductor along with the Hall voltage can be used to measure (a) the sign of the current carriers and (b) the number density of current carriers. Assume a thickness T for the flat conductor.

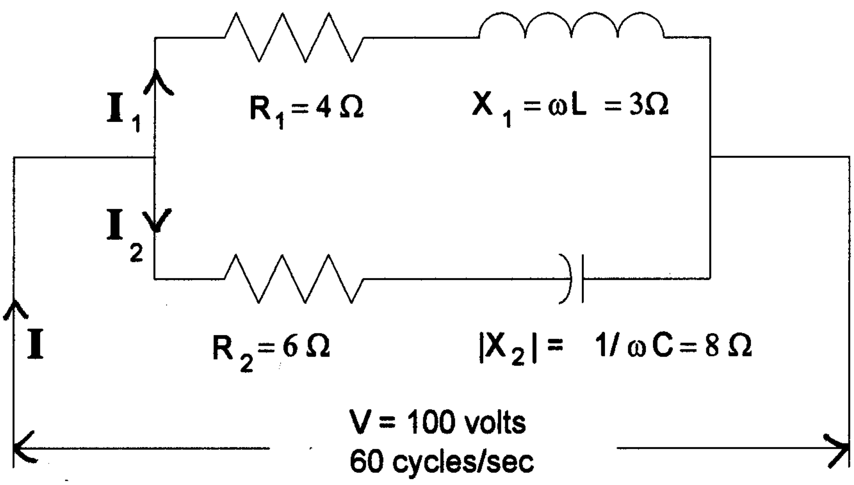


II-4. A 60 cycle AC voltage of 100 volts rms is applied across the circuit shown below where indicated.

a) Find the impedance of the circuit.

b) Find the rms current I.

c) Fully describe the current I_1.



II-5. A nonrelativistic particle of mass *1 GeV/c²* moves in the central potential

$$V(r) = -\frac{1}{r^6} + \frac{0.75}{r^8},$$

where r is in Angstroms, and V is in eV.

a) Find the frequency of small oscillations about the minimum.

b) Estimate the energy of the ground state.

(Take $\hbar = 6.6 \times 10^{-16}\ eV \cdot s$, $c = 3.0 \times 10^{8}\ m/s$ and $1\text{Å} = 10^{-10}\ m$.)

II-6. Provide the best estimate for the following important physical quantities.

1) The rest mass of the electron:

(a) 0.05 MeV/c^2 **(b)** 0.5 MeV/c^2

(c) 5 MeV/c^2 **(d)** 50 MeV/c^2

2) The distance between a Hydrogen atom and the Oxygen atom in a water molecule:

(a) 10^{-5} meters **(b)** 10^{-7} meters

(c) 10^{-8} meters **(d)** 10^{-10} meters

3) The binding energy, per nucleon, of a typical nucleus:

(a) 500 eV **(b)** 5,000 eV

(c) 50,000 eV **(d)** 500,000 eV **(e)** 5,000,000 eV

4) The energy required to remove one electron from a neutral atom:

(a) 0.001 eV **(b)** 0.1 eV

(c) 10 eV **(d)** 1,000 eV

II-7. If a flexible string under tension carries electric current and is placed in a homogeneous magnetic field, the equilibrium profile of the string will be a circular arc lying in the plane perpendicular to the field. The tension will be uniform along the string. Call the tension T, the current I and the field B. What is the radius of that arc?

II-8. A small body of mass density ρ is dropped from rest a height h above the surface of a lake. The density of the water in the lake is ρ_0 and $\rho_0 > \rho$. Neglecting all frictional effects:

a) What is the velocity of the body just before it hits the lake?

b) What is the acceleration of the body at first when in the water?

c) What is the maximum depth d to which the body will sink prior to returning to the surface?