Physics Graduate School Qualifying Examination

August 15, 1996

Part I

Instructions: Work all problems. This is a closed book examination. Start each problem on a new pack of yellow paper and use only one side of each sheet. All problems carry the same weight. Write your student number on the upper right-hand corner of each answer sheet.

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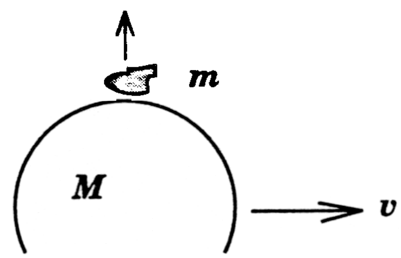
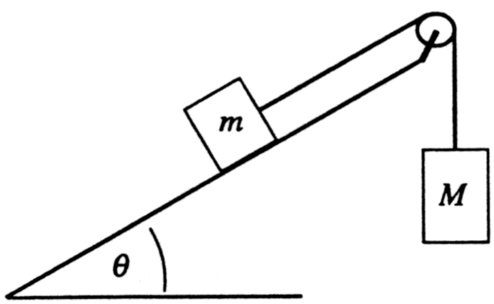
1. An asteroid with mass M, radius r and velocity v is heading for a collision with Earth. Suppose that we are able to place an explosive charge on the asteroid which is capable of blowing out a piece of rock with mass m and an initial speed equal to the escape velocity from the asteroid. The charge is placed so that the rock is ejected radially outward from the center of the asteroid and perpendicular to its direction of motion. Find the ratio m/M such that the asteroid's velocity will be deflected through an angle of 10^{-3} radians relative to its original direction.



2. A mass m, resting on a rough incline, is connected by a massless string to a hanging mass M. The mass m is on the verge of sliding down the incline. Assume that the string does not stretch and that the pulley is massless and frictionless.

   (a) What is the coefficient of static friction between m and the incline?

   (b) How much mass must be added to M so that m will be on the verge of sliding up the incline?



3. When one produces a beam of ions or electrons, the space charge within the beam causes a potential difference between the axis and the surface of the beam. A 10 mA beam of 50 keV protons ($v=3\times10^6$ m/s) travels along the axis of an evacuated beam pipe. The beam has a circular cross section of 1 cm diameter. Calculate the potential difference of the surface of the beam relative to a point on the axis. (Assume the current density is uniform throughout the beam's diameter.)

4. A point charge lies at the center of a dielectric sphere of radius a and dielectric constant ε which is surrounded by vacuum. What is the bound surface charge density at the surface of the dielectric sphere?

5. A student uses a grating spectrometer to measure the wavelength of four strong lines in the visible hydrogen spectrum. The results are:

Color	λ (nm)	$1/\lambda$ (m^{-1})
	656.3	1.524×10^6
	486.1	2.057×10^6
	434.1	2.304×10^6
	410.2	2.438×10^6

Use these data to obtain the value of the Rydberg constant for hydrogen. (This is part of the Balmer Series with $n_f = 2$.)

6. Two rocket ships (S_1 and S_2) approach each other with a relativistic relative speed v. S_1 signals to S_2 with a laser beam of frequency f' as measured in the reference frame of S_1. Derive an expression for the frequency f observed by the crew of S_2.

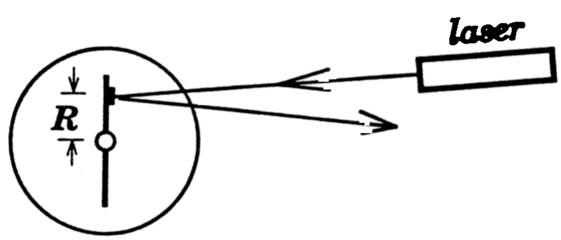
7. The power radiated by a non-relativistic accelerated point charge is given by

$$P = \frac{e^2}{6\pi\varepsilon_0 c^3} a^2$$

Calculate the total energy radiated when an electron moving with velocity v passes a proton at rest with a distance of closest approach b. Assume that v is small compared to c (the speed of light), but large enough that the deflection of the electron as it passes the proton is negligible. The following integral may be helpful:

$$\int_{-\infty}^{+\infty} \frac{dx}{(a^2 + x^2)^2} = \frac{\pi}{}$$

8. To determine the power of a laser (frequency f), scientists use a version of Cavendish's (or Coulomb's) torsion balance. A small mirror is attached to one arm of the device, at a distance R from the rotation axis, and the laser beam is directed so as to hit the mirror perpendicularly. The torque measured is τ. What is the power of the laser—the energy radiated per unit time?



Physics Graduate School Qualifying Examination

August 16, 1996

Part II

Instructions: Work all problems. This is a closed book examination. Start each problem on a new pack of yellow paper and use only one side of each sheet. All problems carry the same weight. Write your student number on the upper right-hand corner of each answer sheet.

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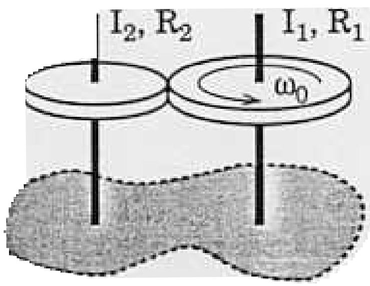
1. Two disks (moments of inertia I_1 and I_2, radii R_1 and R_2) are mounted on frictionless vertical axles so that their edges may be brought into contact. Initially, disk 1 is rotating with an angular velocity ω_0, as shown, and disk 2 is at rest. The disks are then brought into contact, slipping between their edges stops, and they thereafter rotate in contact with each other without slipping. Calculate the final angular velocities of the two disks.



2. The coefficient of restitution is defined as the ratio of the relative velocity of separation to the relative velocity of approach in a collision

$$e = \frac{v_2 - v_1}{u_1 - u_2}$$

e=1 for an elastic collision and e=0 for a completely inelastic collision.

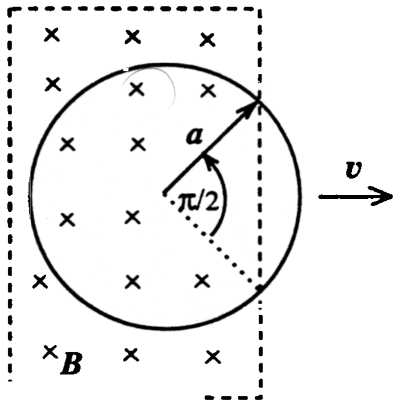
A simple experiment to measure e for one system consists of dropping a steel ball bearing (initially at rest) from a known height h onto a stationary, horizontal glass plate. After a time t, the bearing comes to rest . Obtain a relationship between this time t and the coefficient of restitution.

3. At t=0, the x-component of the B-field in vacuum is given by

$$B_x = A \sin kz \qquad \text{(in rationalized MKS units)}$$

where A and k are real positive constants. All other components of **B** and **E** are initially zero. Find all components of **B** and **E** for a later time t.

4. A wire is formed into a circular loop of radius a and resistance R. It is pulled at constant velocity v out of a region of uniform magnetic field whose magnitude is B. The plane of the loop and the velocity are both perpendicular to the direction of **B**. Assume that the magnetic field strength falls off rapidly at the edge of the region. Find the power in the wire at the instant when the edge of the field region subtends an angle of $\pi/2$ from the center of the loop, as shown.



5. A proton is incident on another proton at rest. What is the minimum kinetic energy of the incident proton needed to produce a proton—anti-proton pair?

$$p + p \longrightarrow p + p + p + \bar{p}$$

Express your answer in terms of E_0, the rest energy of a proton.

6. A particle with energy E is incident from the left on a downward potential step of depth U so that

$$U(x) = 0 \quad \text{for } x \leq 0$$
$$= -U \quad \text{for } x > 0$$

where U is a known positive constant. The resulting wave function is of the form

$$\psi(x) = A e^{ikx} + B e^{-ikx} \qquad \text{for } x \leq 0$$
$$= C e^{2ikx} \qquad \text{for } x > 0$$

where k is a known positive constant.

(a) Calculate the energy E in terms of U for this particle.

(b) What is the reflection coefficient in the previous problem?

7. A mass m is dropped from rest at a height H above a point P on the ground. Assume that Coriolis effects are negligible. Estimate the minimum mean horizontal distance between the ground strike point and P which arises from quantum effects.

8. A point source of ions is placed at the center of one end of an evacuated solenoid of length L, radius R (with $R/L<<1$), and magnetic field B. A small aperture is located symmetrically at the other end of the solenoid. The ions have charge Q, mass m, and non-relativistic speed v. They leave the source at a small angle θ with respect to the axis of the solenoid. For what value(s) of v will the ions exit through the aperture?