Physics Graduate School Qualifying Examination

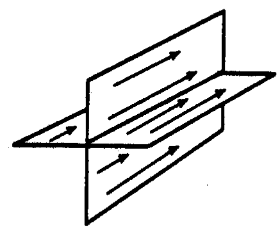
August 19, 1999

Part I

Instructions: Work all problems. This is a closed book examination. Calculators may not be used. Start each problem on a new pack of yellow paper and use only one side of each sheet. All problems carry the same weight. Write your 3-digit student number on the upper right-hand corner of each answer sheet. All sheets which you receive should be handed in, even if blank.

I-1. A straight tunnel connects two points on the earth's surface. A capsule dropped into one end of the tunnel with zero velocity emerges from the other end at a time T later. Assuming that the capsule travels through the tunnel without friction, and that the earth is a uniform, non-rotating sphere, calculate T and show that it is independent of the length of the tunnel.

I-2. Two infinite, thin, conducting planes are perpendicular to each other. Each carries a uniformly distributed surface current density J amps/meter parallel to the line of intersection between the planes. Calculate the resulting magnetic field strength **H** at all points in space.



I-3 Anti-protons (rest mass m) collide with protons at rest. It is hoped to observe production of a pair of particles each with rest mass M. What is the minimum kinetic energy of the anti-proton beam such that the pair production will be energetically possible?

$$p + \overline{p} \to \psi + \overline{\psi}$$

I-4. A wheel is pulled over a frictionless washboard surface with a constant horizontal velocity component. The equation for the surface is $y = A\cos(2\pi x / \lambda)$ where y is the height of the surface, and x is the horizontal distance along the surface. At what value of v_x does the wheel begin to lose contact with the surface, and at what point on the surface will this occur? (assume that the wheel radius $<< \lambda$)

I-5. A non-conducting ring of radius r and mass m carries a uniformly distributed charge q. The ring is set spinning with angular velocity ω about an axis through its center and perpendicular to the plane of the ring. This axis of rotation is observed to precess with period T while maintaining a constant angle θ with the direction of a uniform magnetic field. Express the magnitude of the magnetic field in terms of given quantities.

I-6. Four point charges lie along the z-axis. Two charges $+q$ lie at $z = \pm 2a$. Two charges $-q$ lie at $z = \pm a$. What is the electric field at a general point $\mathbf{r}$ if $r >> a$?

I-7. A particle of mass m, charge q, is in a region of space where the electric field is

$$E_x = E_0 \sin(\omega t - \phi) \text{ and } E_y = E_z = 0.$$

At $t = 0$, the particle is observed to be at rest at the origin. Obtain $\mathbf{r}(t)$ and $\mathbf{v}(t)$ for $t > 0$, and sketch $x(t)$ for $\phi = 0$.

I-8. A positron of unknown energy is found to follow the same trajectory, when traveling perpendicular to a uniform magnetic field, as a 1 KeV proton. What is the kinetic energy of the positron?

Physics Graduate School Qualifying Examination

August 20, 1999

Part II

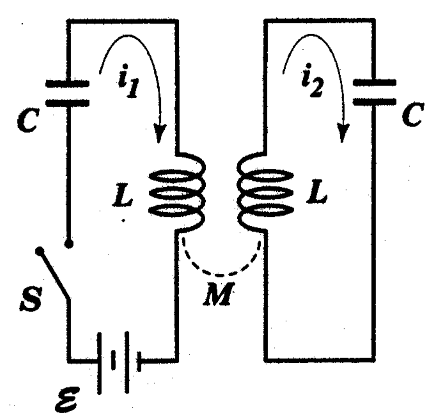
Instructions: Work all problems. This is a closed book examination. Calculators may not be used. Start each problem on a new pack of yellow paper and use only one side of each sheet. All problems carry the same weight. Write your 3-digit student number on the upper right-hand corner of each answer sheet. All sheets which you receive should be handed in, even if blank.

II-1. A certain element is the "daughter" in a radioactive decay, and itself undergoes a subsequent decay. Given that the decay constant of the parent is λ_1, its own decay constant is λ_2, and that at $t = 0$ there are N_0 parent nuclei and no daughters, derive an expression for the number of daughters as a function of time.

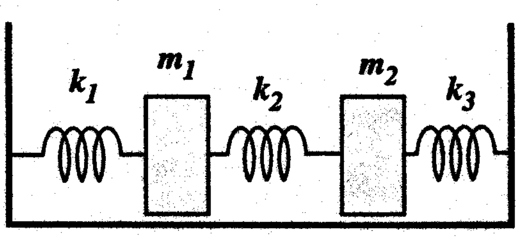
II-2. A very thin uniform circular disc of radius a and mass M spins about an axis through its center which makes an angle α with the normal to the disc. What torque is required to maintain a constant angular velocity ω?

II-3. Two parallel wires, each of radius a, have their centers a distance d apart and carry equal currents in opposite directions. Neglecting the flux within the wires, calculate the inductance per unit length of this pair of wires.

II-4. Consider the simple resistanceless circuit shown at the right in which one suddenly closes the switch at $t = 0$. The source is a battery with constant emf $\mathcal{E}$. An equivalent mechanical system in which two masses, each connected to a rigid wall and to each other by springs, are free to slide on a frictionless surface as shown below.



(a) Set up the differential equation for the currents I_1 and I_2 and show that they have the same form as those for the displacements in the mechanical system.

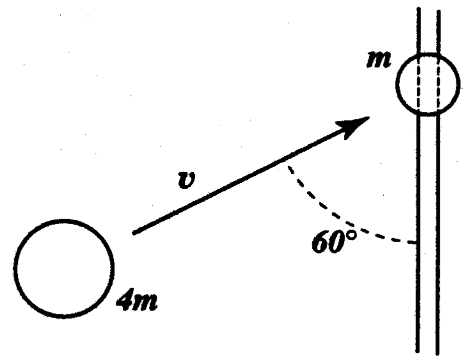


(b) What are the masses m_i and spring constants k_i such that the two systems are exact analogues of each other?

(c) What specific mechanical action is equivalent to closing the switch S in the electrical circuit?

II-5. A photon of wavelength λ collides with a fast electron of energy E traveling in the opposite direction. Calculate the wavelength of the photon scattered in a direction opposite to its incident direction.

II-6. A sphere of mass m is initially at rest but free to slide along a fixed, perfectly smooth rod. Another sphere of mass $4m$, speed v, makes a head-on collision with the first sphere as shown. Assuming that the collision is perfectly elastic, what is the velocity of each sphere immediately after the collision?



II-7. A circular disc of thickness t and conductivity σ rotates about its symmetry axis with an angular velocity ω. A magnetic field of strength B and cross-sectional area A is applied normal to the disc at a distance r from its axis. Calculate the approximate torque felt by the disc. Assume that A is much less than the area of the disc.

II-8. Consider a uniform charge distribution in the form of a sphere of radius a, with two spherical holes of radius b centered at $z = c$ $(b < c < a-b)$ and at $z = -d$ $(b < d < a-b)$.

(a) Find the potential $\phi(\mathbf{r})$ for $r > a$.

(b) Find $\phi(\mathbf{r})$ for $r < a$, but outside the holes.

(c) Find the electric field at the center of the hole located at $z = c$.

(d) Under what conditions will the dipole moment of this system vanish?